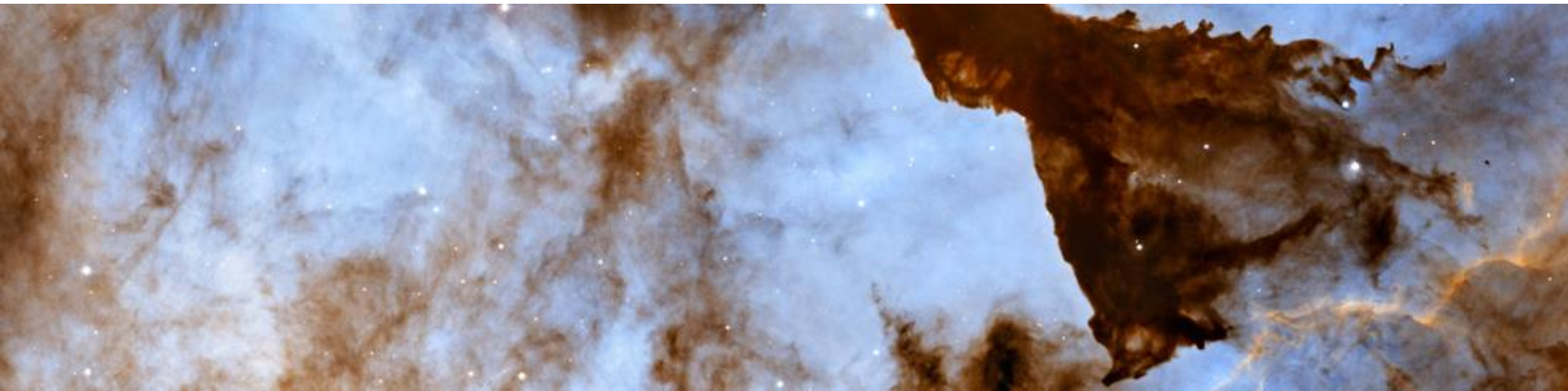


# HLADNI MOLEKULSKI OBLACI



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# Osobine

- najgušća vrsta MZM
- objekti sa sopstvenom gravitacijom
- veoma turbulentni
- prisutno magnetno polje
- akrecija na česticama prašine

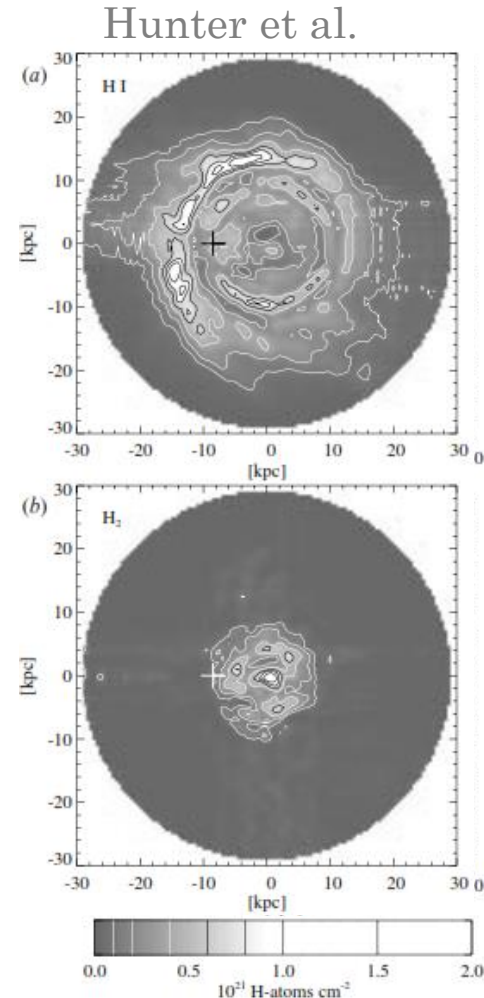


dominantan  
fizički proces?

	<i>Veličina</i>	<i>Masa i gustina</i>	<i>Predstavnici</i>	<i>Karakteristike</i>
Džinovski molekularni oblaci	Oko 50–120 pc	$\approx 10^6 M_{\odot}$ ; $n \approx 10^2\text{--}10^3$ čestica po $\text{cm}^{-3}$	Orionov džinovski molekularni oblak, Maglina Orač	Detektovano je preko 100 različitih molekula
Mali oblaci, globule	Oko 1 sg	$2\text{--}100 M_{\odot}$	Bokove globule	Okolina u H II regionima; sadrže $\text{H}_2$ , CO i He i silikatne čestice
Difuzni vlaknasti oblaci	Do 100 pc	$30$ čestica po $\text{cm}^{-3}$	Infracrveni cirusi	Detektovani na velikim galaktičkim širinama - gusta, amonijačna jezgra.

# Prostorna raspodela MC

- zauzima manje od 1% zapremine međuzvezdanog prostora
  - između 3.5 i 7.5 kpc od centra Galaksije
  - disk debljine 50-75 pc
- položaj usko povezan sa spiralnim granama
  - protivi se teoriji disocijacije ( $t < 10^6$  god)



# CO mapa naše Galaksije

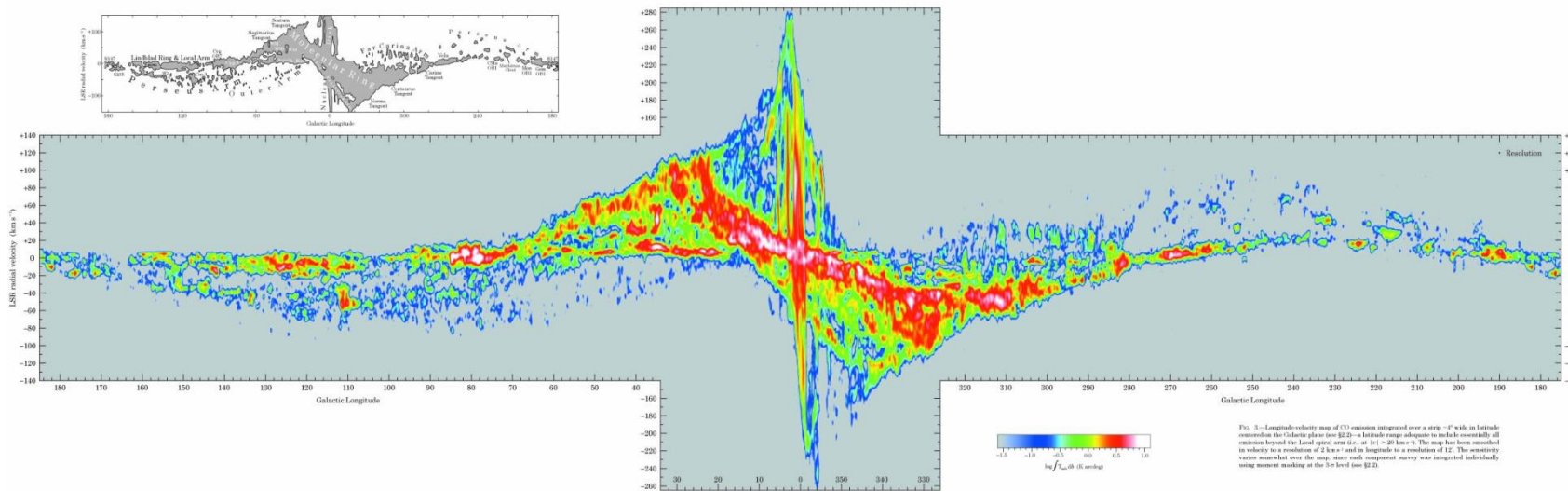
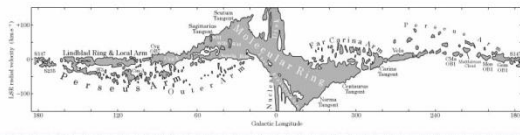
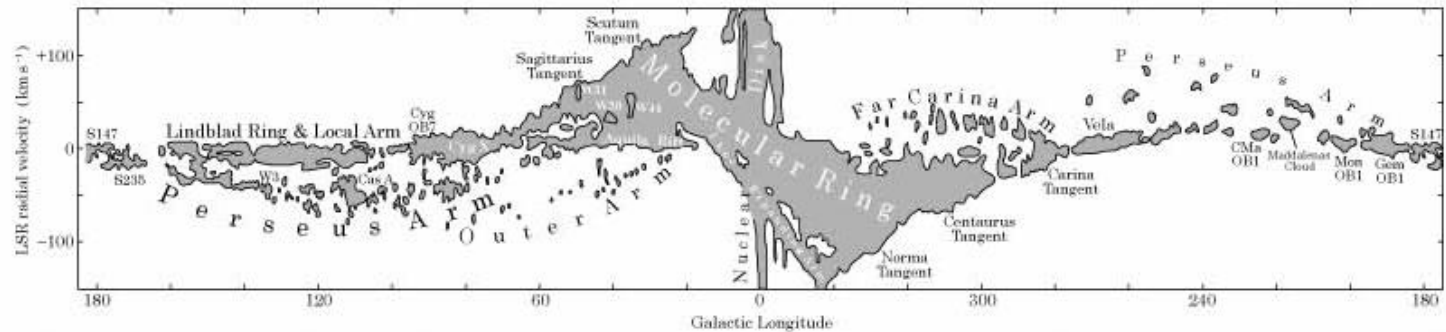
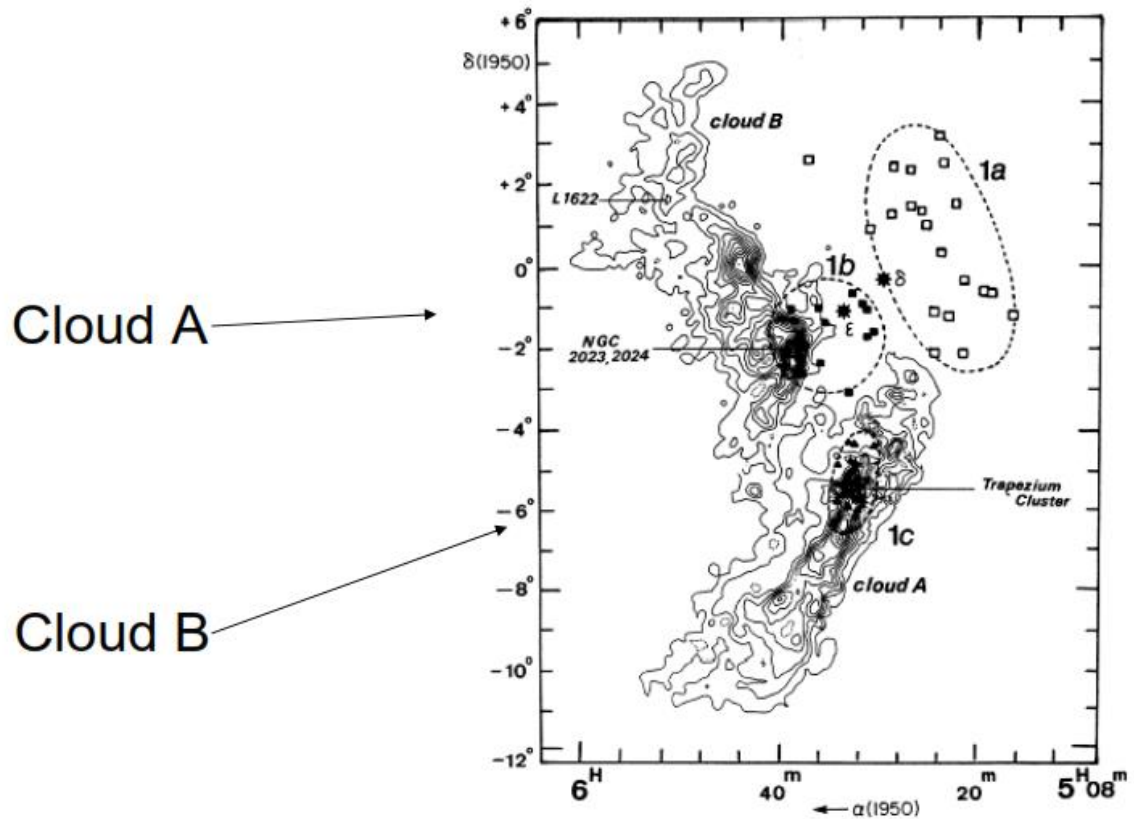
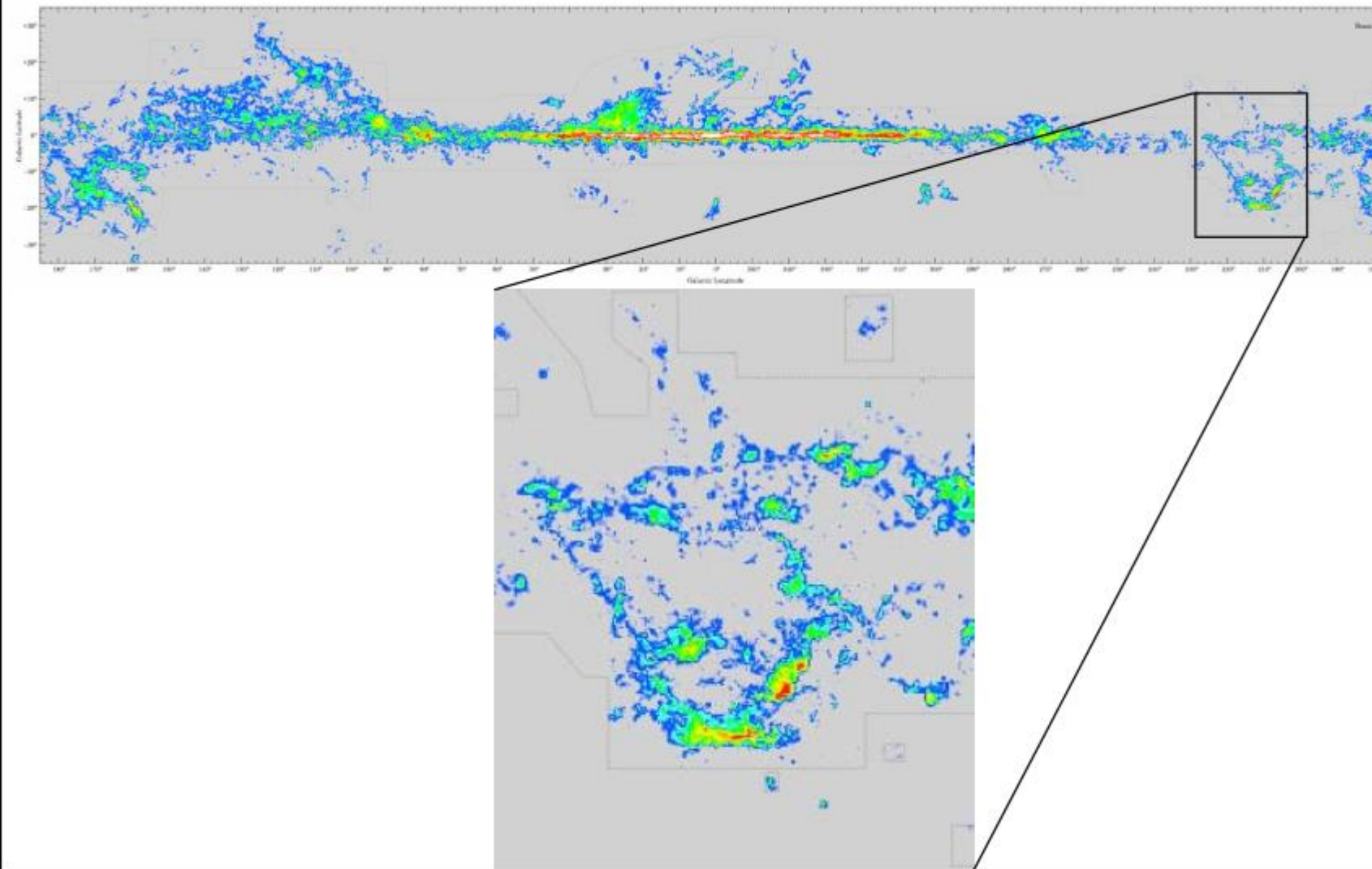


Fig. 3.—Longitude-velocity map of CO emission integrated over a strip  $2^{\circ}$  wide in latitude centered on the Galactic plane (see §2.2). A latitude map analogous to this one contains all emission beyond the Local spiral arm (i.e., at  $|l| > 20$  km s $^{-1}$ ). The map has been smoothed in velocity to a resolution of 2 km s $^{-1}$  and in longitude to a resolution of 12'. The secondary axes summarize over the map, since each component survey was integrated individually using moment stacking at the 3 $\sigma$  level (see §2.2).

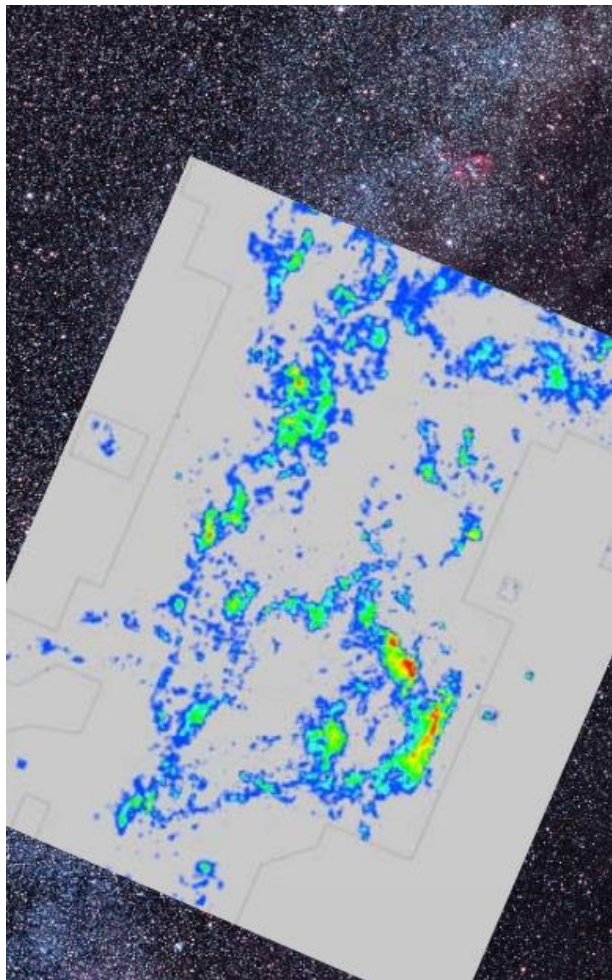
# Primer oblaka Orion A i B



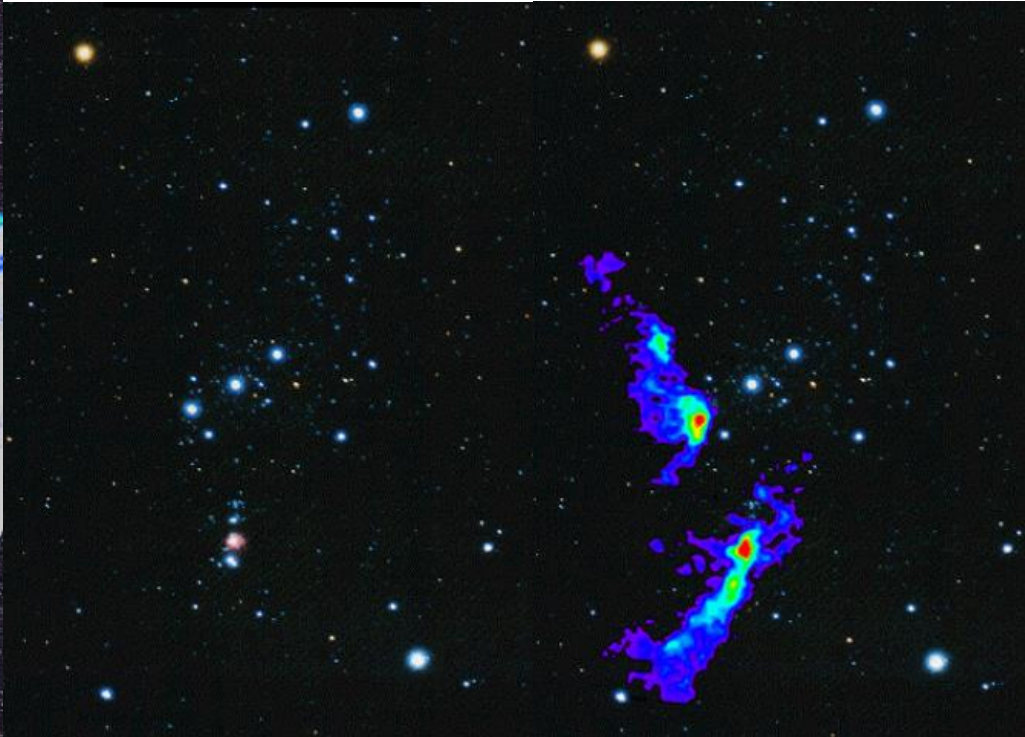
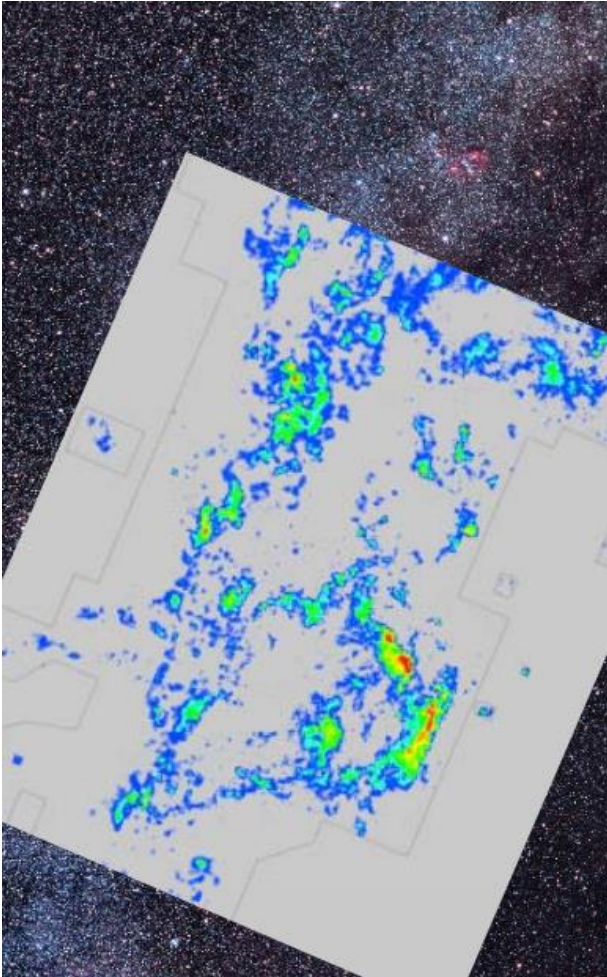




# Simnci u vidljivom i mape CO

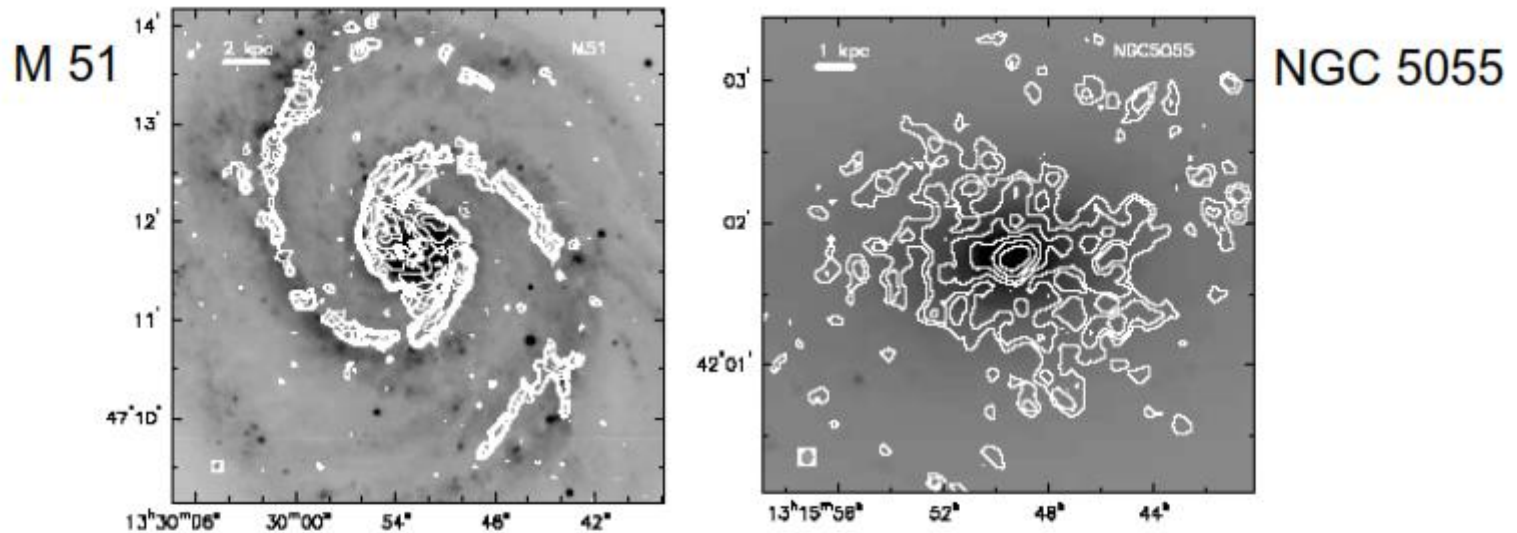


# Simnci u vidljivom i mape CO





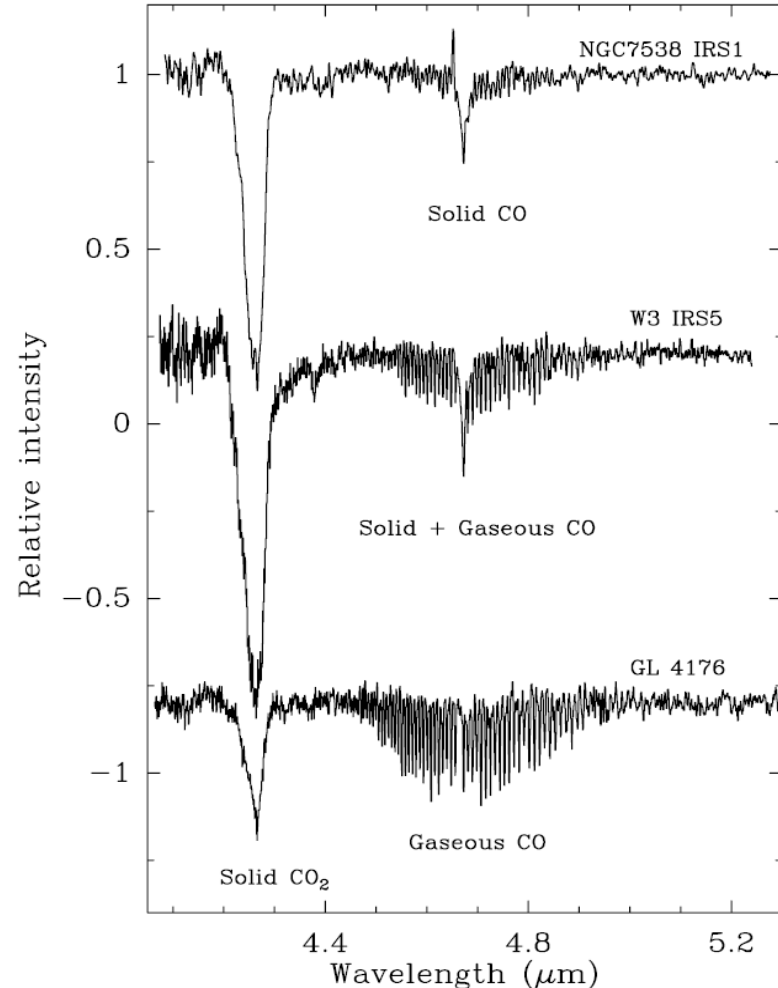
# CO mape drugih galaksija



*Figure 1. Left: CO in M51 from the BIMA Survey of Nearby Galaxies (SONG), overlaid on an optical image of the galaxy. The image contains zero spacing data and thus samples all of the spatial frequencies to the resolution limit (shown by the small box in the lower left). The CO is very strongly concentrated to the spiral arms and lies upstream of most of the ionized gas. Right: BIMA SONG image of the CO in NGC 5055 overlaid on an optical image showing that the galaxy is devoid of large-scale spiral structure at visible wavelengths. Some of the off-nuclear CO peaks are associated with weak spiral arms seen in the near infrared. The difference in CO morphology between these two galaxies is rather striking.*

# Primer spektra CO

- Apsorpcija na spektrima mladih zvezd
  - posmatranja ka tri zvezde
  - spektar različitih agregatnih stanja



# Spisak molekula detektovanih do 2003

2 atoms	3 atoms	4 atoms	5 atoms	6 atoms	7 atoms	8 atoms	$\geq 9$ atoms
Hydrogen family							
H <sub>2</sub>	H <sub>3</sub> <sup>+</sup>						
Carbon family							
C <sub>2</sub>	C <sub>3</sub>	c-C <sub>3</sub> H	C <sub>5</sub>	C <sub>5</sub> H	C <sub>6</sub> H	C <sub>7</sub> H	C <sub>8</sub> H
CH	C <sub>2</sub> H	l-C <sub>3</sub> H	C <sub>4</sub> H	l-C <sub>4</sub> H <sub>2</sub>	CH <sub>3</sub> C <sub>2</sub> H	C <sub>6</sub> H <sub>2</sub>	CH <sub>3</sub> C <sub>4</sub> H
CH <sup>+</sup>	CH <sub>2</sub>	C <sub>2</sub> H <sub>2</sub> CH <sub>2</sub> D <sup>+</sup> ?	l-C <sub>3</sub> H <sub>2</sub> c-C <sub>3</sub> H <sub>2</sub> CH <sub>4</sub>	C <sub>2</sub> H <sub>4</sub>		(C <sub>2</sub> H <sub>6</sub> )	c-C <sub>6</sub> H <sub>6</sub>
Oxygen + hydrogen and/or carbon							
CO	C <sub>2</sub> O	C <sub>3</sub> O	HCOOH	CH <sub>3</sub> OH	HCOCH <sub>3</sub>	HCOOCH <sub>3</sub>	(CH <sub>3</sub> ) <sub>2</sub> O
CO <sup>+</sup>	HCO	HOCO <sup>+</sup>	H <sub>2</sub> C <sub>2</sub> O	HC <sub>2</sub> CHO	c-C <sub>2</sub> H <sub>4</sub> O	CH <sub>3</sub> COOH	CH <sub>3</sub> CH <sub>2</sub> OH
OH	HCO <sup>+</sup> HOC <sup>+</sup> H <sub>2</sub> O (H <sub>2</sub> O <sup>+</sup> ) CO <sub>2</sub>	H <sub>2</sub> CO H <sub>3</sub> O <sup>+</sup>	H <sub>2</sub> COH <sup>+</sup>		CH <sub>2</sub> CHOH	CH <sub>2</sub> OHCHO	(CH <sub>3</sub> ) <sub>2</sub> CO
Nitrogen + hydrogen and/or carbon							
CN	HCN	C <sub>3</sub> N	HC <sub>3</sub> N	CH <sub>3</sub> CN	HC <sub>5</sub> N	CH <sub>3</sub> C <sub>3</sub> N	HC <sub>7</sub> N
NH	HNC N <sub>2</sub> H <sup>+</sup> NH <sub>2</sub>	HCCN HCNH <sup>+</sup> H <sub>2</sub> CN NH <sub>3</sub>	CH <sub>2</sub> CN HC <sub>2</sub> NC H <sub>2</sub> CHN H <sub>2</sub> NCN HNC <sub>3</sub>	CH <sub>3</sub> NC HC <sub>3</sub> NH <sup>+</sup> C <sub>5</sub> N	CH <sub>2</sub> CHCN NH <sub>2</sub> CH <sub>3</sub>		HC <sub>9</sub> N HC <sub>11</sub> N CH <sub>3</sub> CH <sub>2</sub> CN CH <sub>3</sub> C <sub>5</sub> N?

2 atoms	3 atoms	4 atoms	5 atoms	6 atoms	7 atoms	8 atoms	≥9 atoms
Nitrogen + oxygen + hydrogen and/or carbon							
NO	HNO N <sub>2</sub> O	HNCO		NH <sub>2</sub> CHO			NH <sub>2</sub> CH <sub>2</sub> COOH?
Molecules with sulphur							
CS	C <sub>2</sub> S	C <sub>3</sub> S		CH <sub>3</sub> SH			
SO	HCS <sup>+</sup>	HNCS					
SO <sup>+</sup>	H <sub>2</sub> S	H <sub>2</sub> CS					
NS	OCS SO <sub>2</sub>						
Miscellaneous molecules							
SiO	SiCN	SiC <sub>3</sub>	SiC <sub>4</sub>				
SiS	<i>c</i> -SiC <sub>2</sub>		SiH <sub>4</sub>				
HF	MgCN						
SiC	MgNC						
CP	NaCN						
HCl	AlNC						
KCl							
NaCl							
PN							
SiN							
AlF							
AlCl							
SH							
FeO?							

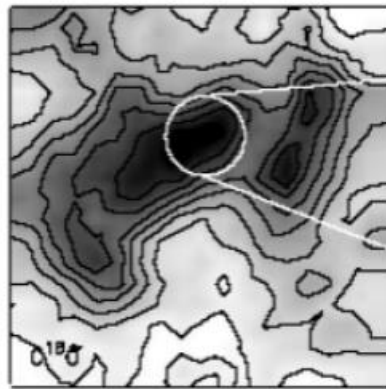


# Struktura MC

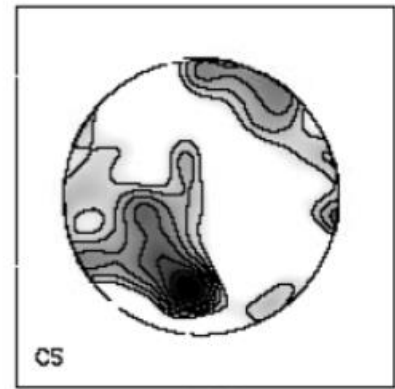
- Gusto jezgro
- Zbijene strukture
- Centralni oblak molekula
- Fotodisocijativni regioni



90''

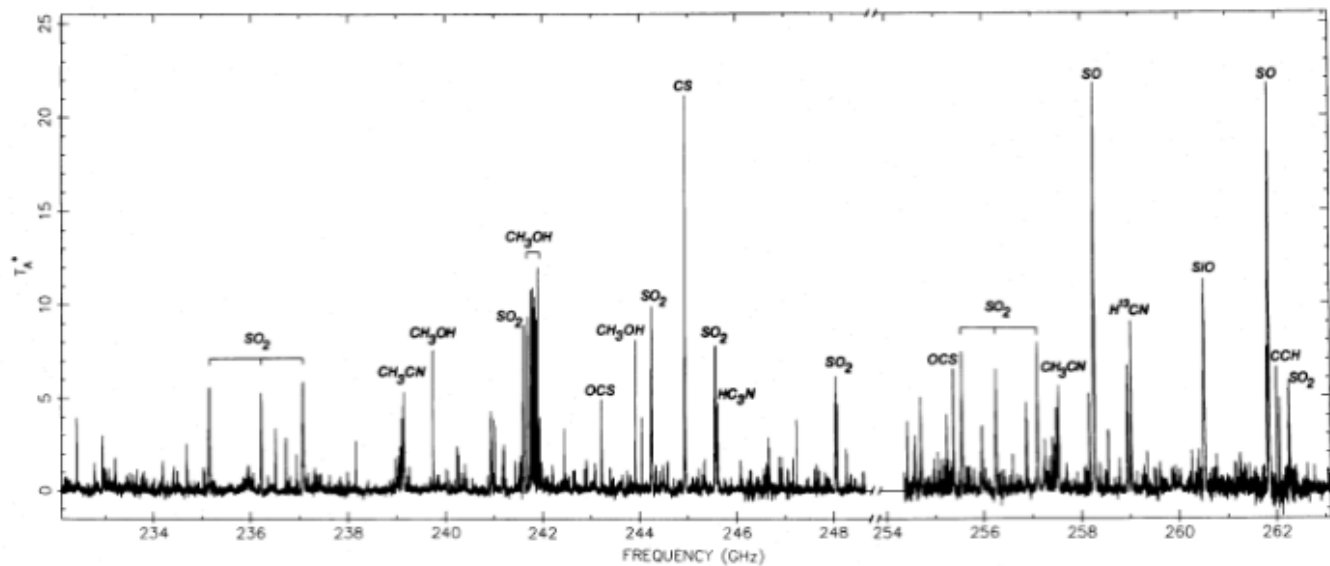
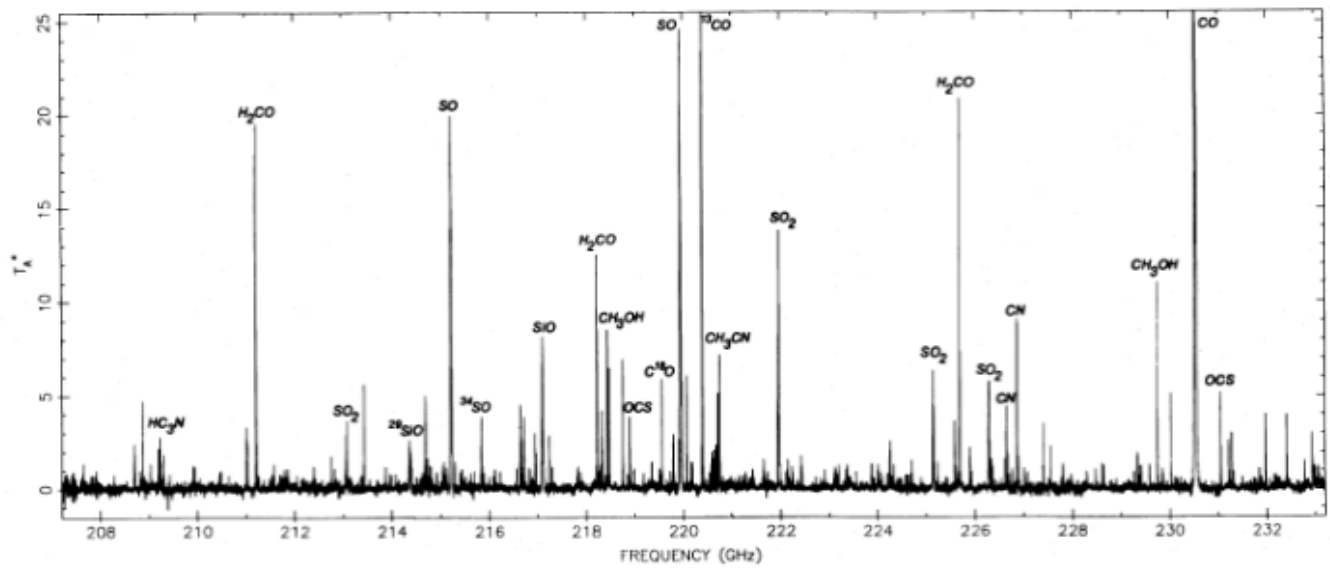


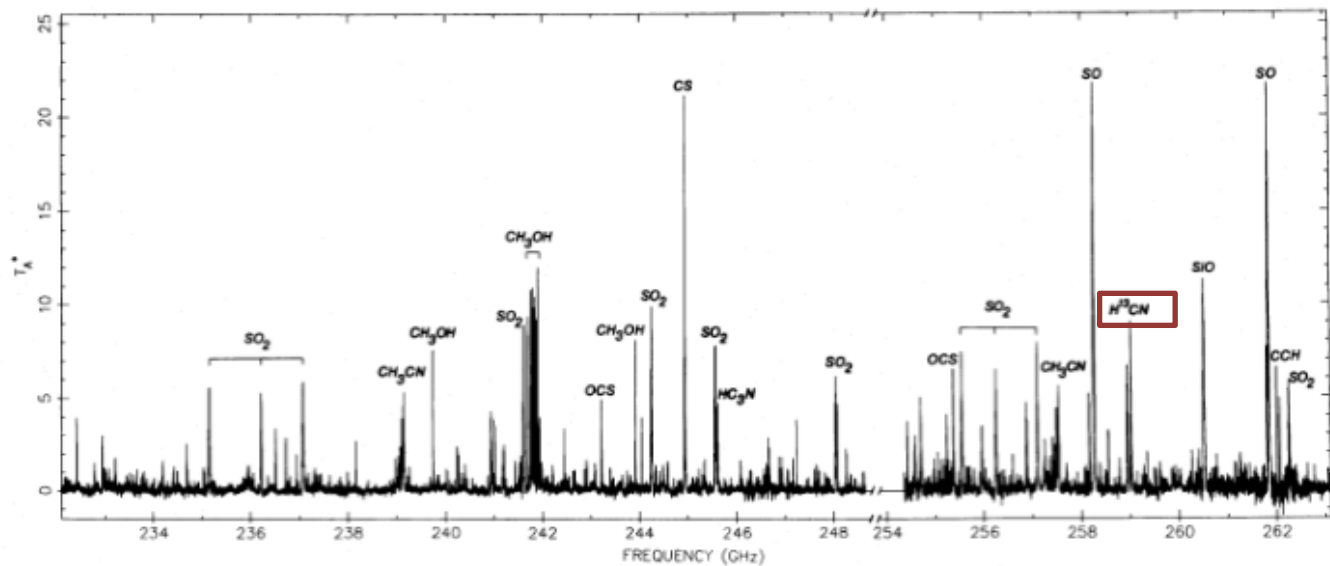
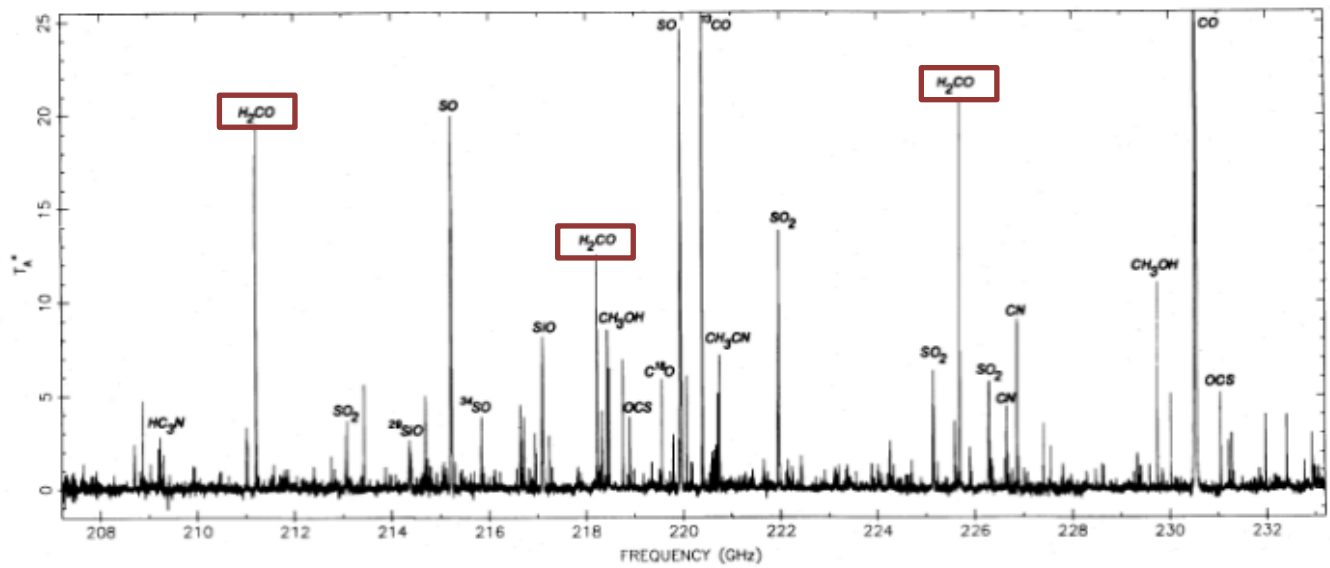
50''



CS

10''





# Markeri gustog gasa

Molecule	Transition	Frequency (GHz)	$E/k$ (K)	$n_{\text{crit}}$ (cm <sup>-3</sup> ) @ 10 K	$n_{\text{eff}}$ (cm <sup>-3</sup> ) @ 10 K
CS	1-0	49.0	2.4	$4.6 \times 10^4$	$7.0 \times 10^3$
	2-1	98.0	7.1	$3.0 \times 10^5$	$1.8 \times 10^4$
	3-2	147.0	14	$1.3 \times 10^6$	$7.0 \times 10^4$
HCO <sup>+</sup>	1-0	89.2	4.3	$1.7 \times 10^5$	$2.4 \times 10^3$
	3-2	267.6	26	$4.2 \times 10^6$	$6.3 \times 10^4$
HCN	1-0	88.6	4.3	$2.6 \times 10^6$	$2.9 \times 10^4$
	3-2	265.9	26	$7.8 \times 10^7$	$7.0 \times 10^5$
H <sub>2</sub> CO	2 <sub>12</sub> -1 <sub>11</sub>	140.8	6.8	$1.1 \times 10^6$	$6.0 \times 10^4$
	3 <sub>13</sub> -2 <sub>12</sub>	211.2	17	$5.6 \times 10^6$	$3.2 \times 10^5$
	4 <sub>14</sub> -3 <sub>13</sub>	281.5	30	$9.7 \times 10^6$	$2.2 \times 10^6$
NH <sub>3</sub>	(1,1)	23.7	1.1	$1.8 \times 10^3$	$1.2 \times 10^3$
	(2,2)	23.7	42	$2.1 \times 10^3$	$3.6 \times 10^4$

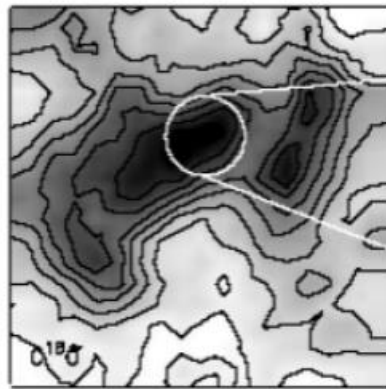


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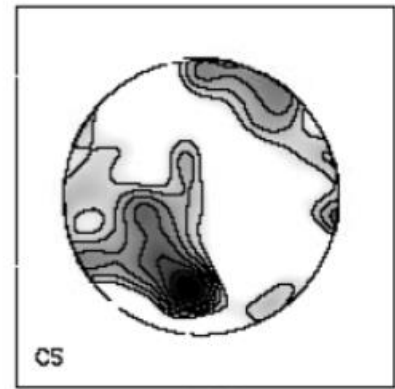
- Gusto jezgro
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90''



50''



CS

10''

# Rotacioni prelazi

- Izraz za energiju:  $E_{\text{rot}} = \frac{1}{2I} (\text{angular momentum})^2$   
 $(\text{angular momentum})^2 = J(J+1) \left(\frac{h}{2\pi}\right)^2$   $J = 0, 1, 2, 3, \dots$   $\Delta J = +1$  or  $-1$

– u slučaju linearnog molekula  $I = \mu r^2$

– možemo to zapisati na sledeći način:

$$E_{\text{rot}} = J(J+1) \left(\frac{h}{2\pi}\right)^2 \frac{1}{2I} + B = h/(8\pi^2 I) \rightarrow E_{\text{rot}}(J) = hBJ(J+1)$$

– ovakav prelazak (u slučaju krutog rotatora) stvara emisiju ili apsorpciju na frekvenciji:

$$\nu = \Delta E/h = B[(J''+1)(J''+2) - J''(J''+1)] = 2B(J''+1)$$

# Vibracioni nivoi

- Za odstupanje  $r - r_e$  važi:  $F = -k(r - r_e)$ .  
(u klasičnoj mehanici)  $V = (1/2)k(r - r_e)^2$ .
- U kvantnoj mehanici:  $E_{\text{vib}}(v) = h\nu_{\text{osc}}(v + 1/2)$   $\nu_{\text{osc}} = 1/(2\pi)\sqrt{(k/\mu)}$   
 $v = 0, 1, 2, 3, \dots$ 
  - za velike vrednosti  $v$  molekuli provode većinu vremena daleko od ravnotežnog položaja
  - privilegovano širenje molekula
  - promena u vrednosti  $B$ , odnosno  $I$ 
    - » *centrifugalna sila*  
 $\mu\omega^2 r = \mu (\text{angular momentum}/I)^2 r = J(J + 1)h^2/(4\pi^2)1/(\mu r^3)$
    - » *ravnoteža?*

# Vibracioni nivoi

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(u klasičnoj mehanici)  $V = (1/2)k(r - r_e)^2$ .

- U kvantnoj mehanici:  $E_{\text{vib}}(v) = hv_{\text{osc}}(v + 1/2)$   $v_{\text{osc}} = 1/(2\pi)\sqrt{(k/\mu)}$   
 $v = 0, 1, 2, 3, \dots$

- Usled promena  $r \rightarrow$  promena  $I = \mu r^2$   
 – izraženo za velika  $v$ ,  $J$  (za hladne molekule nije bitno)

$$k(r - r_e) = \frac{J(J + 1) \left(\frac{h}{2\pi}\right)^2}{\mu r^3}$$

$$= \frac{J(J + 1) \left(\frac{h}{2\pi}\right)^2}{\mu r_e^3 \left(1 + \frac{r - r_e}{r_e}\right)^3}$$

$$\text{total energy} = \frac{J(J + 1) \left(\frac{h}{2\pi}\right)^2}{2\mu r^2} - \frac{1}{2}k(r - r_e)^2$$

$$= \frac{J(J + 1) \left(\frac{h}{2\pi}\right)^2}{2\mu r_e^2 \left(1 + \frac{r - r_e}{r_e}\right)^2} - \frac{1}{2}k(r - r_e)^2$$

$$\simeq hBJ(J + 1) - \frac{J^2(J + 1)^2 h^4}{32\pi^4 \mu^2 r_e^6 k}$$



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- Usled promena  $r \rightarrow$  promena  $I = \mu r^2$

– izraženo za velika  $v$ ,  $J$  (za hladne molekule nije bitno)

$$(r - r_e)/r_e \ll 1$$

$$k(r - r_e) = \frac{J(J + 1) \left(\frac{h}{2\pi}\right)^2}{\mu r^3}$$

$$= \frac{J(J + 1) \left(\frac{h}{2\pi}\right)^2}{\mu r_e^3 \left(1 + \frac{r - r_e}{r_e}\right)^3}$$

$$\text{total energy} = \frac{J(J + 1) \left(\frac{h}{2\pi}\right)^2}{2\mu r^2} - \frac{1}{2}k(r - r_e)^2$$

$$= \frac{J(J + 1) \left(\frac{h}{2\pi}\right)^2}{2\mu r_e^2 \left(1 + \frac{r - r_e}{r_e}\right)^2} - \frac{1}{2}k(r - r_e)^2$$

$$\approx hBJ(J + 1) - \frac{J^2(J + 1)^2 h^4}{32\pi^4 \mu^2 r_e^6 k}$$

# Vibracioni nivoi

- Za odstupanje  $r - r_e$  važi:  $F = -k(r - r_e)$   
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 $v = 0, 1, 2, 3, \dots$
- Usled promena  $r \rightarrow$  promena  $I = \mu r^2$   
 – izraženo za velika  $v$ ,  $J$  (za hladne molekule nije bitno)  $(r - r_e)/r_e \ll 1$

$$\begin{aligned} \text{total energy} &= \frac{J(J+1)\left(\frac{h}{2\pi}\right)^2}{2\mu r^2} - \frac{1}{2}k(r - r_e)^2 \\ &= \frac{J(J+1)\left(\frac{h}{2\pi}\right)^2}{2\mu r_e^2\left(1 + \frac{r - r_e}{r_e}\right)^2} - \frac{1}{2}k(r - r_e)^2 \\ &\simeq hBJ(J+1) - \frac{J^2(J+1)^2 h^4}{32\pi^4 \mu^2 r_e^6 k} \end{aligned}$$

$$E = hBJ(J+1) - hDJ^2(J+1)^2 + \dots$$

$$D = 16\pi^2 B^3 \mu/k = 4B^3/\nu_{\text{osc}}^2$$

$$v = 2B(J'' + 1) - 4D(J'' + 1)^3$$

$$B_v = B_e - \alpha(v + 1/2) + \dots \quad D_v = D_e - \beta(v + 1/2)$$

# Verovatnoća?

- verovatnoća prelaza je data sa:  $A_{ji} = \frac{64\pi^4}{3hc^3 4\pi\epsilon_0} \nu^3 |d|^2$

– dipolni momenat:  $d = e \int \Phi_j r \Phi_i dV$

$$d^2 = f(J) (e \int \phi_i r \phi_i dV)^2 = \frac{(J+1)}{(2J+3)} d_p^2$$

– verovatnoća:  $A_{ji} = 1.046 * 10^{21} \nu^3 d_p^2 \frac{(J+1)}{(2J+3)}$

– particiona funkcija:

$$\begin{aligned}
 U &= \sum_J (2J+1) \exp\left[-\frac{hBJ(J+1)}{kT_{\text{ex}}}\right] \simeq \int_0^\infty (2J+1) \exp\left[-\frac{hBJ(J+1)}{kT_{\text{ex}}}\right] dJ \\
 &\simeq \int_0^\infty \exp\left[-\frac{hBJ'}{kT_{\text{ex}}}\right] dJ', \quad \text{with } J' = J^2 + J \\
 &\simeq \frac{kT_{\text{ex}}}{hB}
 \end{aligned}$$

# Verovatnoća?

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– verovatnoća:  $A_{ji} = 1.046 * 10^{21} \nu^3 d_p^2 \frac{(J+1)}{(2J+3)}$

– particiona funkcija:  $U = (kT_{ex}) / (hB) + 1/3$

$$\begin{aligned}
 U &= \sum_j (2J+1) \exp\left[-\frac{hBJ(J+1)}{kT_{ex}}\right] \simeq \int_0^\infty (2J+1) \exp\left[-\frac{hBJ(J+1)}{kT_{ex}}\right] dJ \\
 &\simeq \int_0^\infty \exp\left[-\frac{hBJ'}{kT_{ex}}\right] dJ', \quad \text{with } J' = J^2 + J \\
 &\simeq \frac{kT_{ex}}{hB}
 \end{aligned}$$

# Verovatnoća?

- verovatnoća prelaza je data sa:  $A_{ji} = \frac{64\pi^4}{3hc^3 4\pi\epsilon_0} \nu^3 |d|^2$

CO u osnovnom stanju  
nema nuklearni spin  
a ugaoni momenat = 0



veoma jednostavan slučaj

$$U = \sum_j$$

$$\approx \int_0^{\infty} \exp\left[-\frac{hBJ'}{kT_{\text{ex}}}\right] dJ', \quad \text{with } J' = J^2 + J$$

$$\approx \frac{kT_{\text{ex}}}{hB}$$

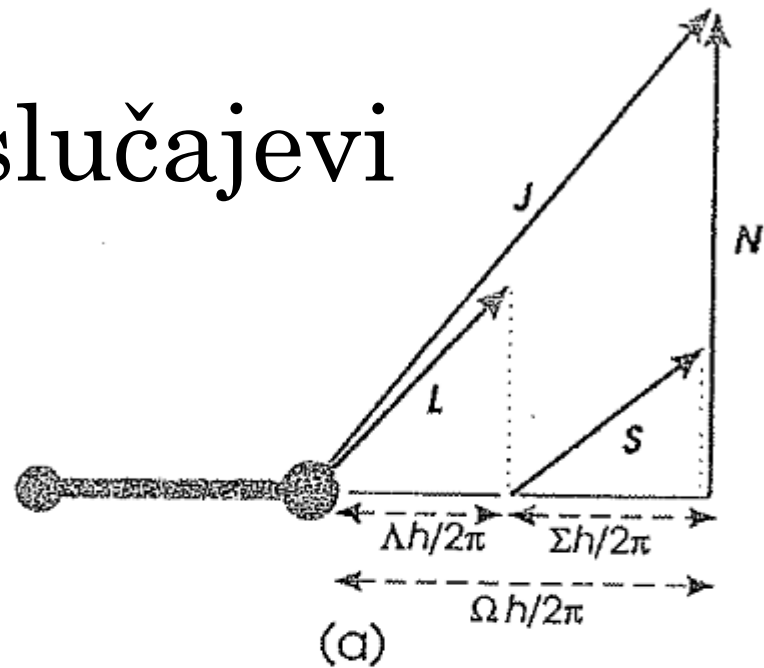
1)  
3)  $d^2$

Šta utiče na energiju i intenzitet rotacionih i vibracionih linija?

dužina molekula, svedena masa  
dipolni momenat, ose simetrije  
superpozicija kvantnih stanja,  
prostorna orijentacija molekula...

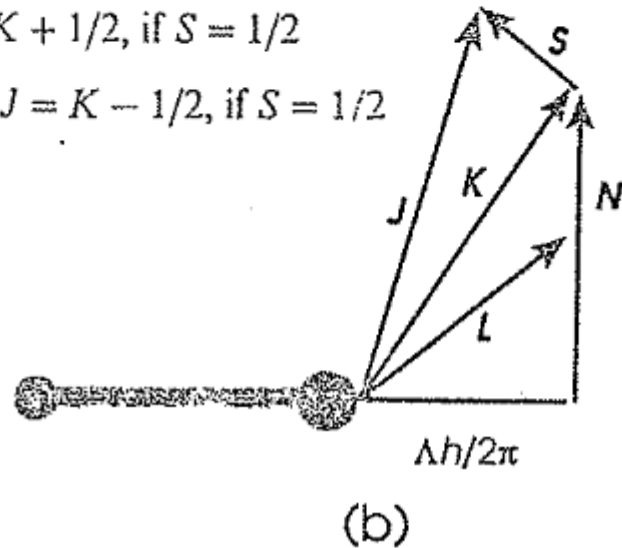
# Hundovi slučajevi

- Linearan molekul
- Slaganje spinova



$$\text{energy} = hBK(K + 1) + \text{constant} * K \quad \text{for } J = K + 1/2, \text{ if } S = 1/2$$

$$\text{energy} = hBK(K + 1) - \text{constant} * (K + 1) \quad \text{for } J = K - 1/2, \text{ if } S = 1/2$$



# Ne-linearni molekuli

- Uopšten slučaj

$$E = \frac{1}{2}I_x\omega_x^2 + \frac{1}{2}I_y\omega_y^2 + \frac{1}{2}I_z\omega_z^2 = \frac{(\text{angular Momentum about } x)^2}{2I_x} + \frac{(\text{angular Momentum about } y)^2}{2I_y} + \frac{(\text{angular Momentum about } z)^2}{2I_z}$$

- U sistemu gde je jedna od osa odabrana za osu molekula:  $E = hBJ(J + 1) + h(A - B)\Omega^2$

$$\longleftarrow A = h/(8\pi^2 I_z)$$

$$(\text{angular momentum about } x)^2 + (\text{angular momentum about } y)^2$$

$$= N^2 = J^2 - \Omega^2 = (J(J + 1) - \Omega^2) [h/(2\pi)]^2$$

- Kvadrpolni član takođe utiče na linije
  - za izražen magnetni momenat



Šta utiče na energiju i intenzitet rotacionih i vibracionih linija?

dužina molekula, svedena masa  
dipolni momenat, ose simetrije  
superpozicija kvantnih stanja,  
prostorna orijentacija molekula...

# Oblik molekula

- Momenti inercija duž glavnih osa su obeleženi u skladu sa konvencijom:

$$I_c \geq I_b \geq I_a$$

- Simetrični

- elektronski prelasci dozvoljeni
- rotacioni i vibracioni nisu

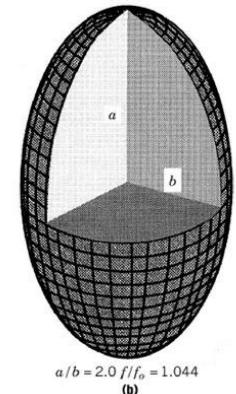
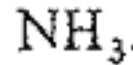
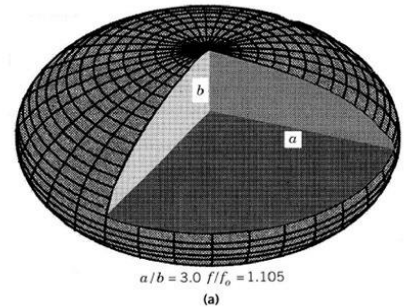
- Sferno simetrični

- Kružno simetrični (Symetric Top)

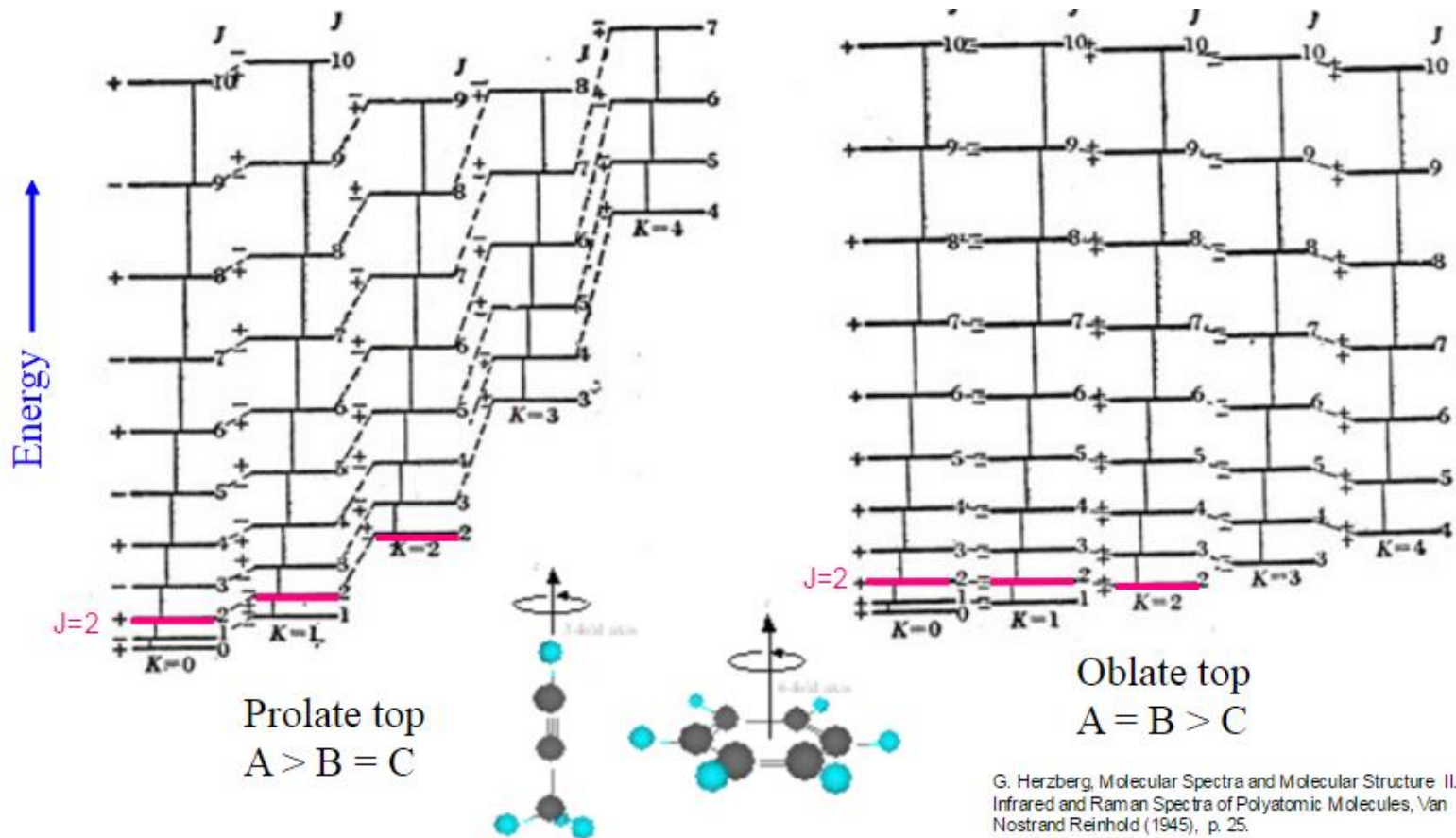
- Oblate (a)  $I_c > I_b = I_a$

- Prolate (b)  $I_c = I_b > I_a$

- Asimetrični



# Spektar Prolate/Oblate molekula



Oblate: benzene,  $C_6H_6$ , ammonia,  $NH_3$

Prolate: chloromethane,  $CH_3Cl$ , propyne,  $CH_3C\equiv CH$

# Spektar

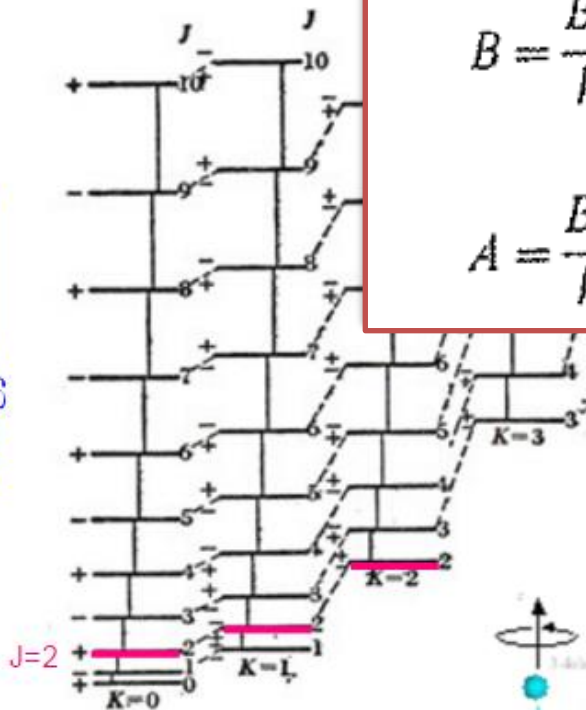
# molekula

$$E(J, K) = hBJ(J + 1) + h(A - B)K^2$$

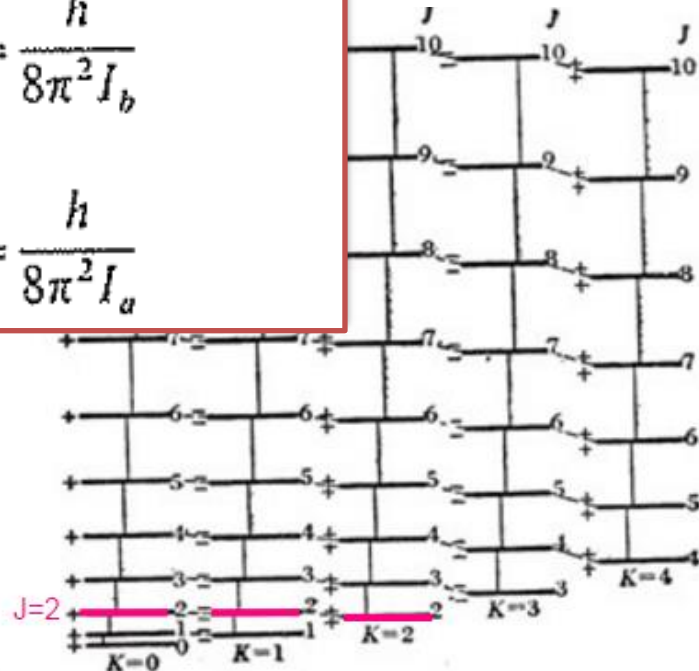
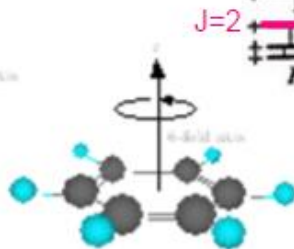
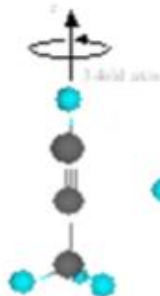
$$B = \frac{B'_b}{h} \left( \frac{h}{2\pi} \right)^2 = \frac{h}{8\pi^2 I_b}$$

$$A = \frac{B'_a}{h} \left( \frac{h}{2\pi} \right)^2 = \frac{h}{8\pi^2 I_a}$$

Energy ↑



Prolate top  
 $A > B = C$



Oblate top  
 $A = B > C$

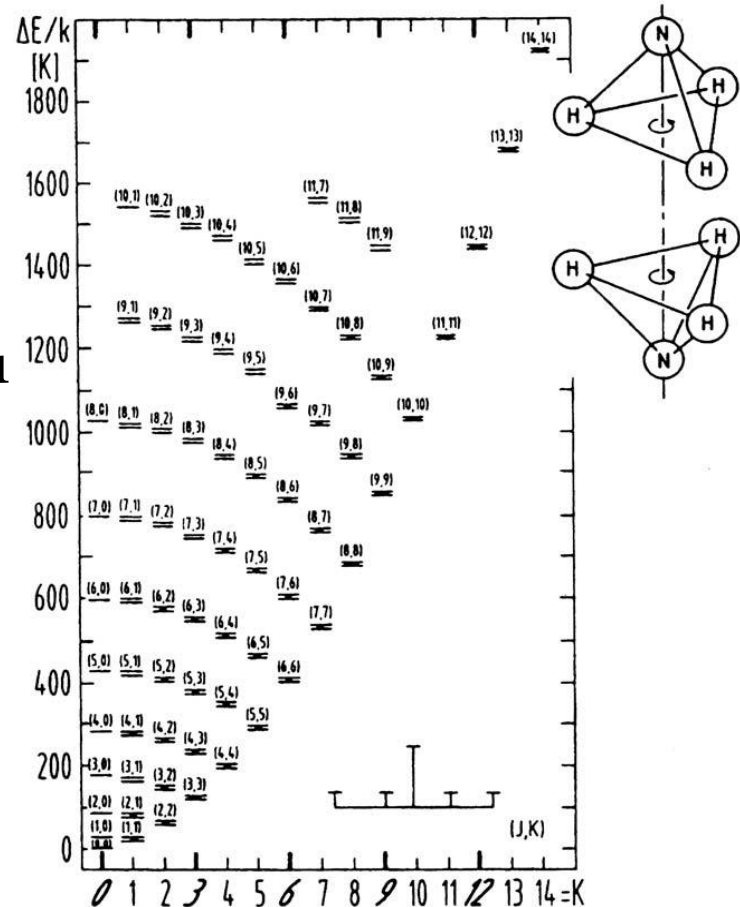
G. Herzberg, Molecular Spectra and Molecular Structure. II. Infrared and Raman Spectra of Polyatomic Molecules, Van Nostrand Reinhold (1945), p. 25.

Oblate: benzene,  $C_6H_6$ , ammonia,  $NH_3$

Prolate: chloromethane,  $CH_3Cl$ , propyne,  $CH_3C\equiv CH$

# Orto i Para forme

- Orto – paralelni nuk. spin
- Para – anti-paralelan spin
- Prelazak iz jedne u drugu formu je izričito zabranjen prelaz
  - detekcija  $H_2$
  - temperatura formiranja molekula



# Jačina linija molekula – primer na CO

- Kako odrediti
  - Temperaturu?
  - Linijsku gustinu?
  - CO ima više prelaza i može biti sačinjen od više izotopa
    - niz linija
      - raspodela po energetskim nivoima
      - potvrđuju/opovrgavaju LTE
      - daju uvid u T sredine
      - optički guste/tanke
    - niz frekvencija u radio, IC itd.
      - CMB ili drugi pozadinski izvor

# Temperatura sjaja

$$\begin{aligned}\int T_B dv &= \frac{c^2 h}{8\pi k} \frac{A_{ji} g_j}{g_i} \frac{1}{v} \frac{g_i}{U} \exp\left[-\frac{E}{kT_{\text{ex}}}\right] (N_{\text{TOT}} \rho R) \\ &= \frac{c^2 h}{8\pi k} \frac{A_{ji} (2J+3)}{(2J+1)} \frac{1}{2B(J+1)} \frac{hB}{kT_{\text{ex}}} (2J+1) \exp\left[-\frac{hBJ(J+1)}{kT_{\text{ex}}}\right] (N_{\text{TOT}} \rho R) \\ &= \frac{c^2 h^2}{16\pi k^2} \frac{1}{T_{\text{ex}}} A_{ji} \frac{(2J+3)}{(J+1)} \exp\left[-\frac{hBJ(J+1)}{kT_{\text{ex}}}\right] (N_{\text{TOT}} \rho R)\end{aligned}$$

# Temperatura sjaja

$$\begin{aligned}
 \int T_B dv &= \frac{c^2 h}{8\pi k} \frac{A_{ji} g_j}{g_i} \frac{1}{v} \frac{g_i}{U} \exp\left[-\frac{E}{kT_{\text{ex}}}\right] (N_{\text{TOT}} \rho R) \\
 &= \frac{c^2 h}{8\pi k} \frac{A_{ji} (2J+3)}{(2J+1)} \frac{1}{2B(J+1)} \frac{hB}{kT_{\text{ex}}} (2J+1) \exp\left[-\frac{hBJ(J+1)}{kT_{\text{ex}}}\right] (N_{\text{TOT}} \rho R) \\
 &= \frac{c^2 h^2}{16\pi k^2} \frac{1}{T_{\text{ex}}} \underline{A_{ji}} \frac{(2J+3)}{(J+1)} \exp\left[-\frac{hBJ(J+1)}{kT_{\text{ex}}}\right] (N_{\text{TOT}} \rho R)
 \end{aligned}$$



$$\begin{aligned}
 \int T_B dv &= \frac{4\pi^3 h}{3ck^2(4\pi\epsilon_0)} \frac{v^3}{T_{\text{ex}}} d_p^2 \exp\left[-\frac{hBJ(J+1)}{kT_{\text{ex}}}\right] (N_{\text{TOT}} \rho R) \\
 &= \frac{32\pi^3 h}{3ck^2(4\pi\epsilon_0)} \frac{B^3(J+1)^3}{T_{\text{ex}}} d_p^2 \exp\left[-\frac{hBJ(J+1)}{kT_{\text{ex}}}\right] (N_{\text{TOT}} \rho R)
 \end{aligned}$$



# Temperatura sjaja

$$\int T_B dv = \frac{c^2 h}{8\pi k} \frac{A_{ji} g_j}{g_i} \frac{1}{v} \frac{g_i}{U} \exp\left[-\frac{E}{kT_{\text{ex}}}\right] (N_{\text{TOT}} \rho R)$$

$$= \frac{c^2 h}{8\pi k} \frac{A_{ji} (2J+3)}{(2J+1)} \frac{1}{2B(J+1)} \frac{hB}{kT_{\text{ex}}} (2J+1) \exp\left[-\frac{hBJ(J+1)}{kT_{\text{ex}}}\right] (N_{\text{TOT}} \rho R)$$

$$= \frac{c^2 h^2}{16\pi k^2} \frac{1}{T_{\text{ex}}} \underline{A_{ji}} \frac{(2J+3)}{(J+1)} \exp\left[-\frac{hBJ(J+1)}{kT_{\text{ex}}}\right] (N_{\text{TOT}} \rho R)$$



Koliko je optički gusta sredina?

$$\int T_B dv = \frac{4\pi^3 h}{3ck^2(4\pi\epsilon_0)} \frac{v^3}{T_{\text{ex}}} d_p^2 \exp\left[-\frac{hBJ(J+1)}{kT_{\text{ex}}}\right] (N_{\text{TOT}} \rho R)$$

$$= \frac{32\pi^3 h}{3ck^2(4\pi\epsilon_0)} \frac{B^3(J+1)^3}{T_{\text{ex}}} d_p^2 \exp\left[-\frac{hBJ(J+1)}{kT_{\text{ex}}}\right] (N_{\text{TOT}} \rho R)$$

# Linije različitih izotopa

- Odnos između nekoliko izotopa C

$$N(^{12}\text{CO}) \simeq 76 N(^{13}\text{CO})$$

$$N(^{12}\text{CO}) = 560 N(\text{C}^{18}\text{O})$$

- Za dugo-talasnu aprox. ove linije imaju istu:

- temperaturu pozadinskog zračenja  $T_B = T_{\text{ex}}(1 - e^{-\tau})$

- temperaturu ekscitacije  $T_{\text{ex}}$

- Stoga važi:  $\frac{T_B(^{12}\text{C}^{16}\text{O})}{T_B(^{13}\text{C}^{16}\text{O})} = \frac{(1 - e^{-\tau(^{12}\text{C}^{16}\text{O})})}{(1 - e^{-\tau(^{13}\text{C}^{16}\text{O})})}$

$$N(^{12}\text{C})/N(^{13}\text{C}) = R,$$

$$\tau(^{12}\text{C}^{16}\text{O}) = R\tau(^{13}\text{C}^{16}\text{O})$$

optički tanak sl.

$$\tau = T_B/T_{\text{ex}}$$

$$T_B(^{13}\text{C}^{16}\text{O})/T_B(^{12}\text{C}^{18}\text{O}) = N(^{13}\text{C})/N(^{12}\text{C}) \cdot N(^{16}\text{O})/N(^{18}\text{O})$$

$$\int \tau_\nu dV = \frac{8\pi^3 B d_p^2 N_{\text{TOT}} \rho R (J+1) e^{-hBJ(J+1)/kT_{\text{ex}}}}{3k4\pi\epsilon_0 T_{\text{ex}} + \frac{hB}{3k}} (1 - e^{-h\nu/kT_{\text{ex}}})$$

# Linije različitih izotopa

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optički tanak sl.

$$\tau = T_B/T_{\text{ex}}$$

$$N_{\text{TOT}}\rho R(^{12}\text{C}^{16}\text{O}) = 2.39 \times 10^{15} \frac{e^{hB_{12}J(J+1)/kT_{\text{ex}}}}{J+1} \frac{T_{\text{ex}} + 0.93}{1 - e^{-hv_{12}/hT_{\text{ex}}}} \int \tau dV \text{m}^{-2}$$

$$N_{\text{TOT}}\rho R(^{13}\text{C}^{16}\text{O}) = 2.49 \times 10^{15} \frac{e^{hB_{13}J(J+1)/kT_{\text{ex}}}}{J+1} \frac{T_{\text{ex}} + 0.89}{1 - e^{-hv_{13}/hT_{\text{ex}}}} \int \tau dV \text{m}^{-2}$$

# Prelazi na različit nivo

- Unutar istog molekula od istih izotopa posmatramo prelaskе na (niži) nivo  $l$ ,  $m$

$$\frac{\tau_V(m \text{ to } m+1)}{\tau_V(l \text{ to } l+1)} = \frac{(J_m + 1) e^{-E_m/kT_{cs}} (1 - e^{-h\nu_m/kT_{cs}})}{(J_l + 1) e^{-E_l/kT_{cs}} (1 - e^{-h\nu_l/kT_{cs}})}$$

- Ukoliko su nivoi uzastopni ( $m=l+1$ )

$$\frac{\tau_V(l+1 \text{ to } l+2)}{\tau_V(l \text{ to } l+1)} = \frac{J_l + 2 (1 - e^{-h\nu_m/kT_{cs}})}{J_l + 1 (e^{h\nu_l/kT_{cs}} - 1)}$$

# Detaljnije – sa pozadinskim zračenjem

$$T'_B = c^2/(2kv^2)I_\nu$$

- Prisutnost pozadinskog zračenja ( $T=2.7\text{K}$ )

$$T_B = T_{\text{ex}}(1 - e^{-\tau}) + T_{\text{BCK}}e^{-\tau} \quad \text{or} \quad T_{\text{OBS}} = T_B - T_{\text{BCK}} = (T_{\text{ex}} - T_{\text{BCK}})(1 - e^{-\tau})$$

- Ukoliko predstavimo preko Planka

$$I_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT_{\text{BCK}}} - 1} e^{-\tau_\nu} + \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT_{\text{ex}}} - 1} (1 - e^{-\tau_\nu})$$

$$T'_{\text{OBS}} = T'_B - T'_{\text{BCK}} = \frac{h\nu}{k} \left[ \frac{1}{e^{h\nu/kT_{\text{ex}}} - 1} - \frac{1}{e^{h\nu/kT_{\text{BCK}}} - 1} \right] (1 - e^{-\tau_\nu})$$

$$T'_{\text{OBS}}(1-0) = 5.532 \left( \frac{1}{[e^{5.532/T_{\text{ex}}} - 1]} - 0.1479 \right) (1 - e^{-\tau_\nu}) \quad \text{for } ^{12}\text{C}^{16}\text{O}$$

$$= 5.289 \left( \frac{1}{[e^{5.289/T_{\text{ex}}} - 1]} - 0.1642 \right) (1 - e^{-\tau_\nu}) \quad \text{for } ^{13}\text{C}^{16}\text{O}$$

# Rezultat razmatranja

- Ako uzmemo da je temperatura ekscitacije nekoliko puta veća od pozadinske dobijamo:

$$N_{\text{TOT}}\rho R = 4.3 \times 10^{14} \frac{(T_{\text{ex}} + 0.93)}{e^{-5.53/T_{\text{ex}}}} \int \frac{T'_{\text{OBS}}\tau_V}{(1 - e^{-\tau_V})} dV$$

# Molekul sa dva nivoa

- Nalazi se u polju zračenja  $J_\nu$  sa  $T_B$

$$S_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT_{ex}} - 1} = \frac{\int \phi_\nu J(T_R)_\nu + \varepsilon B_\nu(T_K)}{1 + \varepsilon}$$

$$\varepsilon = \frac{C_{ji}}{A_{ji}} (1 - e^{-h\nu/kT_K}) \quad \rightarrow \quad \varepsilon = \frac{C_{ji}}{A_{ji}} \frac{h\nu}{kT_K}$$

- Uzimamo nekoliko aprox.

$$\int \phi_\nu J_\nu d\nu = J_\nu \int \phi_\nu d\nu = J_\nu(T_R)$$

$$S_\nu = (2k\nu^2)/c^2 T_{ex}$$

$$J_\nu = (2k\nu^2)/c^2 T_R$$

$$B_\nu = (2k\nu^2)/c^2 T_K$$

- Dobijamo:  $T_{ex} = \frac{T_R + \varepsilon T_K}{1 + \varepsilon}$

- Odnosno...

$$T_{ex} = T_K \left( \frac{T_0 + T_B}{T_0 + T_K} \right) \quad T_0 = \frac{h\nu}{k} \frac{C_{ul}}{A_{ul}}$$

# Razmatranje molekula sa dva nivoa

- Kakav je oblik jednačine koju smo dobili?

$$T_{ex} = T_K \left( \frac{T_0 + T_B}{T_0 + T_K} \right) \quad T_0 = \frac{h\nu}{k} \frac{C_{ul}}{A_{ul}}$$

- Ukoliko su sudari dominantni:
  - $T_0$  je veliko i  $T_{ex} \sim T_K$  (ravnoteža sudarima/LT)
- Ukoliko sudari nisu dovoljno česti:
  - $T_0$  je malo i  $T_{ex} \sim T_B$  (ravnoteža zračenja)

- Bez dugotalasne aproksimacije

$$\frac{1}{e^{h\nu/kT_{ex}} - 1} = \frac{1}{1 + \varepsilon} \frac{1}{[e^{h\nu/kT_R} - 1]} + \frac{\varepsilon}{1 + \varepsilon} \frac{1}{[e^{h\nu/kT_K} - 1]}$$

$$\frac{T_0}{T_{ex}} = \frac{T_0}{T_K} + \ln_e \left( \frac{e^{T_0/T_R} + \frac{C_{ji}}{A_{ji}} [e^{T_0/T_R} - 1]}{e^{T_0/T_K} + \frac{C_{ji}}{A_{ji}} [e^{T_0/T_R} - 1]} \right)$$

Pojam kritične gustine



# Procesi u MC

- Hladenje gasa
  - Rotacioni prelazi i emisione linije CO i drugih molekula
  - Sudari molekula sa hladnim česticama prešine
  - Emisija sudarno pobuđenih linija [C ii], [Si ii] [C i]
  - Rotaciono-vibracione linije molekulskog vodonika
- Zagrevanje gasa (OB zvezde, difuzno polje zračenja)
  - Fotoelektricni mehanizam zagrevanja
  - UV-fluorescentno zagrevanje (Ly-Weiber fotoni)

# Određivanje mase MC

- Džinsov zakon daje:

$$\mathcal{M}_J = \frac{4\pi}{3} R_J^3 \rho_c \approx 18 M_\odot T^{1.5} n^{-0.5}$$

- Na osnovu teoreme virijala:

$$\mathcal{M}_{\text{vir}} \approx \Delta v^2 \left( \frac{R_c}{G} \right) \quad \Delta v = \sqrt{\frac{G \mathcal{M}_c}{R_c}}$$

- Na osnovu posmatranja CO:

$$L_{\text{CO}} = D^2 \int I_{\text{CO}} d\Omega \Leftrightarrow L_{\text{CO}} = T_A^*(\text{CO}) \Delta v \pi R_c^3.$$

- **Konačna relacija:**

$$L_{\text{CO}} = T_A^*(\text{CO}) \left( \frac{3\pi G}{4\rho_c} \right)^{1/2} \mathcal{M}_c.$$

# Odnos vodonika i CO?

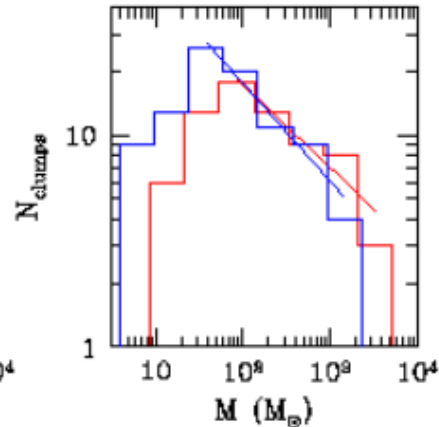
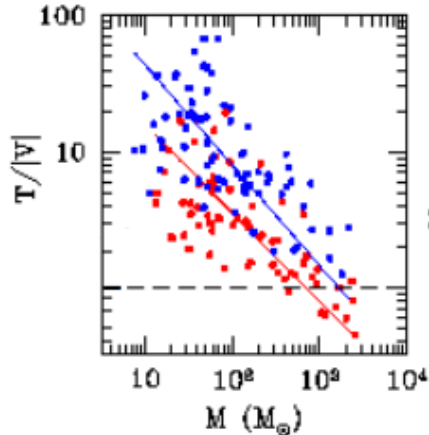
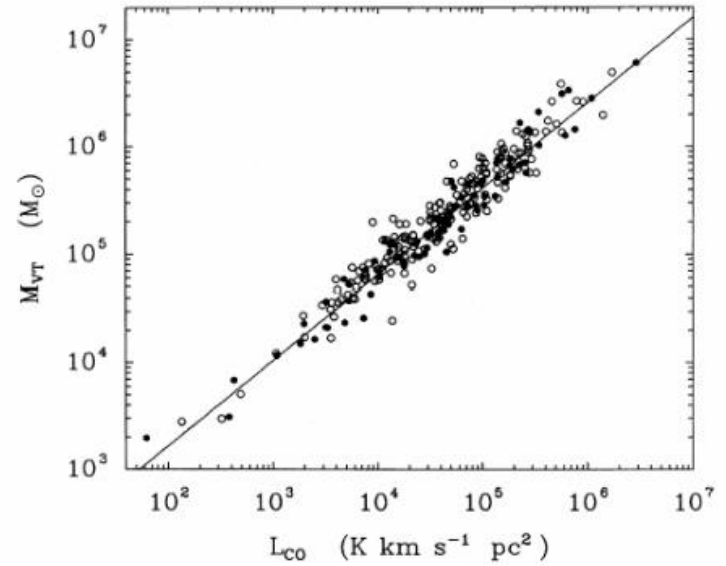
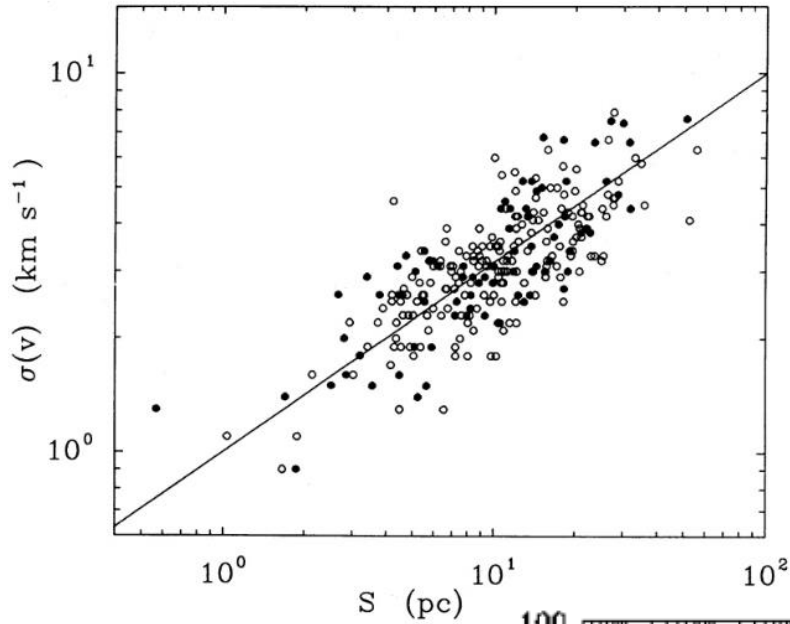
- Odnos se smatra konstantnim – određen je **X faktorom** u skladu sa formulom:

$$\frac{N(\text{H}_2)}{I_{\text{CO}}} = X \cdot 10^{20} \text{ cm}^{-2} [\text{K km s}^{-1}]$$

- Vrednosti dobijene za X variraju

Early work	2-5
$\gamma$ -rays Hunter et al. 1997	1.56 $\pm$ 0.05
HI/IRAS/CO Dame et al. 2001	1.8 $\pm$ 0.3
$A_V$ Lada et al. 2003	$\sim$ 4

# Emprijski zakoni



# Ključna pitanja

Ostaje nekoliko važnih tema koje nisu razjašnjene

- *Da li se GMC formiraju u spiralnim granama galaksija?*
- *Koji od svih procesa (sudari manjih molekulskih struktura, nestabilnost odnosa gravitacija - termalni gas, galaktički udarni talasi/najčešće supernova ostaci, magnetna polja) je najdominantniji u stvaranju molekulskih oblaka?*
- *Da li su sve površinske gustine u GMC iste ili bar približno iste?*
- *Možemo li strukturu GMC-a uopštititi kao diskretnu?*
- *Da li postoji veza između fenomena dugog „životnog veka” spiralnih galaksija sa eventualnim dugim vremenom života molekulskih oblaka koji su zastupljeni u njima?*